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Dynamics of the Bose-Hubbard model

Bogdan Damski (LANL)
and F. M. Cucchietti, J. Dziarmaga, W. H. Zurek

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One dimensional Bose-Hubbard model

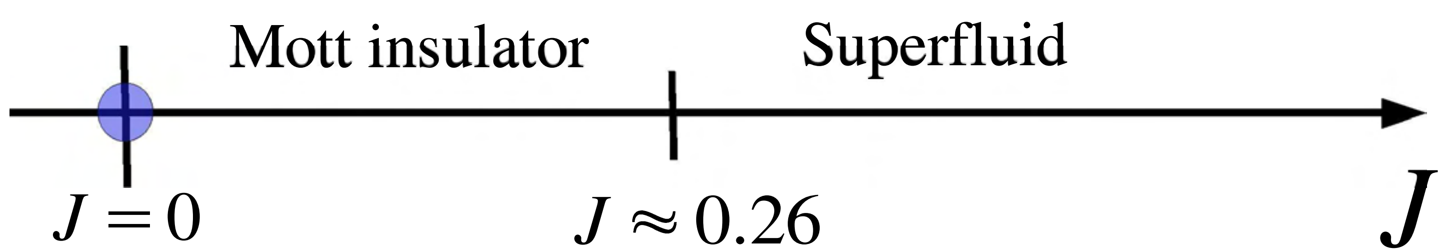
$$\widehat{H} = -J\sum_{i}(\hat{a}_{i+1}^{\dagger}\hat{a}_{i} + h.c.) + \frac{1}{2}\sum_{i}\hat{n}_{i}(\hat{n}_{i}-1)$$

$$J = \frac{t}{\tau_{Q}}$$

1 atom at 2^{nd} lattice site

Evolution starts at t=0 and J=0 from state: $|1,1,1,1,...\rangle$

and proceeds to J>> 1



Nearest neighbor correlation function

$$C = \frac{1}{2} \langle \hat{a}_1^\dagger \hat{a}_0 + h.c. \rangle$$

measurable from momentum distribution of atoms

Fast evolutions ($\tau_Q \ll 1$):

$$\frac{C(t)}{\sqrt{\tau_Q}} = \frac{2}{3}\left(\frac{t}{\sqrt{\tau_Q}}\right)^3 \quad \text{for} \quad t \leqslant \sim \sqrt{\tau_Q}$$

$$C(t \to +\infty) = const \cdot \sqrt{\tau_Q}$$

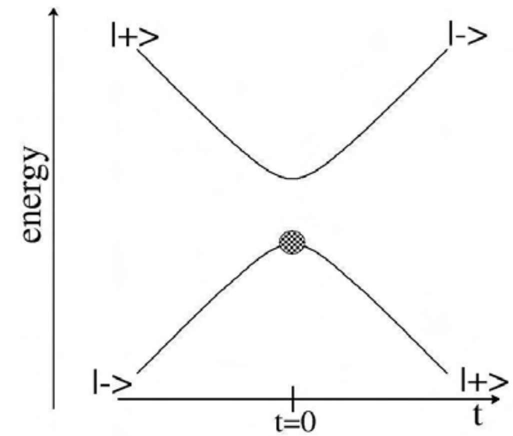
Slow evolutions ($\tau_Q \gg 1$): $C(t \to +\infty) = 1 - \frac{const}{\tau_Q^2}$

The two site/atom system

$$|+\rangle \sim |1,1\rangle + \frac{|0,2\rangle + |2,0\rangle}{\sqrt{2}} \qquad |-\rangle \sim |1,1\rangle - \frac{|0,2\rangle + |2,0\rangle}{\sqrt{2}}$$

Landau-Zener model: evolution starts from anti-crossing center!

$$H = \frac{1}{2}\begin{bmatrix} \frac{4t}{\tau_Q} & 1 \\ 1 & -\frac{4t}{\tau_Q} \end{bmatrix}$$



$$C\,(t \to +\infty) = \frac{1}{2}\langle\, \hat{a}_1^\dagger \hat{a}_0 + h.c.\,\rangle = 1 - 2 \cdot p_{ex}$$

excitation probability of Landau-Zener model

$\tau_Q \ll 1:$

$$C\,(t \to +\infty) = \frac{\sqrt{\pi}}{4}\sqrt{\tau_Q}$$

$\tau_Q \gg 1:$

$$C\,(t \to +\infty) = 1 - \frac{8}{\tau_Q^2} \Rightarrow p_{ex} = \frac{4}{\tau_Q^2} \text{ vs. } e^{-\frac{\pi \tau_Q}{8}}$$

M site/atom model: zero order approximation

A prediction:

$$\frac{C(t)}{\sqrt{\tau_Q}} = \frac{2}{3}\left(\frac{t}{\sqrt{\tau_Q}}\right)^3 \qquad \text{for} \qquad t < \sim \sqrt{\tau_Q}$$

can be obtained from

$$|\psi(t)\rangle = a(t)\,|1,1,1,...\rangle$$
$$+\, b(t)\big(|1,1,..,0,2,1,1,..\rangle + |1,1,..,2,0,1,1,..\rangle + ...\big)/\sqrt{2M}$$

system size

where a(t) and b(t) follow Landau-Zener dynamics starting from anticrossing center.

Summary

Time-dependent correlation functions building up during transitions from Mott insulator to Superfluid regime satisfy characteristic scaling relations.

We were able to characterize build up of correlations by using: variational wave functions, BCS-Bogolubov approach, Kibble-Zurek estimations, and numerical calculations.

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